Structure of Matter - I
April 06, 2017

PROBLEM 1. On electronic structure [12 pnts]

Consider a Au^{6+} ion ($....5d^5$).

a) Calculate the angle between the angular momentum vector and the xy plane for a single 5d electron with m=-1. [2 pnts]
b) Sketch the radial part of this 5d wave function (rR_{5d}). [2 pnts]
c) Calculate the binding energy of hydrogen-like N^{6+}(5d). [1 pnt]
d) Explain why the ionization potential of Au^{6+} is much higher, higher, equal, lower or much lower than N^{6+}(5d). [2 pnt]
e) Determine the ground term and level of Au^{6+}. [3 pnts]
f) Due to the nuclear spin the ground level splits up into 6 hyperfine levels. Determine the possible values of the nuclear spin I. [2 pnt]

PROBLEM 2. On nuclear structure [12 ptn]

Consider the chromium isotope $^{53}_{24}Cr$.

a) Calculate the neutron density [in atomic mass units per fm^3] of the nucleus. [1 pnt]
b) Determine the nuclear spin J and the parity of the nucleus. [3 pnts]
Hint: Use the generic sequence of nuclear shell filling: 1s, 1p, 1d, 2s, 1f, 2p, 1g, ...
c) In reality the nuclear spin of this isotope is 3/2 what does this imply with respect to the energy sequence of the nuclear shells. [2 pnts]
d) There also exists a Cr isotope with nuclear spin I=7/2. What are the possible masses (in amu) of that isotope. Explain your answer. [2 pnts]
e) The $^{53}_{24}Cr$ isotope may be created via the β^- decay of a $^{?}_{?}V$ isotope. Give the complete formula of the decay reaction. Explain your answer. [2 pnts]
f) Finally, assume that the $^{53}_{24}Cr$ isotope consists fully out of antimatter (antiprotons and antineutrons). Give the nuclear spin J and the parity of this antimatter nucleus. Explain your answer. [2 pnts]

PROBLEM 3. Elementary particles [13 pnts]

Consider a charmed baryon with quark content udc decaying into $p + K^- + \pi^+$.

Hint: quark compositions of π^+ : $u\bar{d}$ and K^- : $s\bar{u}$

a) Determine the hypercharge and the azimuthal isospin of this baryon. [2 pnt]

b) Verify that the conservation laws for lepton and baryon number are respected. [1 pnt]

c) Which conservation law is violated? [1 pnt]

d) By which force(s) is the decay driven? [1 pnt]

e) What is the approximate time scale of the decay? [1 pnt]

f) Consider the J=3/2 family of charmed baryons. The 6 baryons of this family are: udc, usc, dsc, uuc, ddc, and ssc. Explain why the existence of charmed baryons with two equivalent quarks (e.g. uuc) requires the additional set of color quantum numbers? [2 pnts]

g) For the π^+, what is the color of the $\bar{d}$ quark if the color of the u quark is blue? [1 pnts]

h) A ρ^+ meson has the same quark content as a π^+. Explain why i) the π^+ (and of course the ρ^+ meson) are much heavier than their individual quarks and ii) why ρ^+ is heavier than π^+ although it contains the same two quarks. [2 pnts]

i) Are these mesons eigenstates of the charge conjugation operator. Explain your answer. [2 pnts]

PROBLEM 4. Magnetic field effects [8 pnts]

a) Show that the $^4D_{1/2}$ level does not split up in a magnetic field. [1 pnt]

Hint: $g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$

b) Explain what this tells us about the alignment of the magnetic moment μ_J. [2 pnt]

c) Sketch the binding energy shifts of the $^4D_{3/2}$ states for an increasing, but still weak magnetic field B (Zeeman effect), indicate the relevant quantum numbers. [2 pnt]

d) The fine structure splitting between the $^4D_{3/2}$ and $^4D_{1/2}$ levels is 5 cm^{-1}. Determine (a sophisticated estimation of) the value of the magnetic field B at which the weak magnetic field approximation is not valid anymore. [2 pnt]

Hint: Use $\mu_B = 0.47\ cm^{-1}/T$

e) How should the system be described in a stronger magnetic field and the energies calculated. [1 pnt]